Reg. No. :

Question Paper Code : 41303

MECH – II

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

Common to Mechanical Engineering (Sandwich) Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Civil Engineering/Civil Engineering and Computer Based Construction/Computer Science and Engineering/Computer and Communication Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/ Electronics and Instrumentation Engineering/Environmental Engineering/ Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Material Science and Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Metallurgical Engineering/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/B.E./B.Tech. (Common to all Branches except Marine Engg.)/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom and Textile Technology/Industrial Bio Technology/Information Technology/Leather Technology/ Petrochemical Technology/ Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Rubber and Plastics Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion Technology)/Textile Technology (Textile Chemistry)

(Regulations 2013)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions.

PART – A — **(10×2=20 Marks)**

1. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$ and hence find the eigen values of A^{-1}.

2. Discuss the nature of the quadratic form $2x^2 + 2xy + 3y^2$.

3. State the necessary condition for the convergence of series of positive terms.

4. Define absolutely convergent and conditionally convergent of a series.

5. Find the curvature of the curve $2x^2 + 2y^2 + 5x - 2y + 1 = 0$.

6. List two important properties of the evolute.

7. If $x = r^2 - \theta^2$, $y = 2r\theta$ find $\frac{\partial r}{\partial x}$.

8. When is a function said to be stationary at a point (x, y) ?

9. Evaluate $\int_{-1}^{2}\int_{x}^{x+2} dydx$.

10. Evaluate $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^4 r^3 \sin\theta \, dr d\theta d\phi$.

PART – B (5×16=80 Marks)

11. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$ and hence find A^{-1}. (10 + 6)

(OR)

b) Reduce the following quadratic form to a canonical form by orthogonal reduction and find the rank, index signature and the nature of the quadratic form : $(-x^2 + y^2 + 4yz + 4zx)$. (8+2+2+2+2)

12. a) i) Use integral test to check the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{x^n + x^{-n}}$ (8)

ii) Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$ by D'Alembert's Ratio test. (8)

(OR)

b) i) Discuss the convergence of the series $\frac{5}{2} - \frac{7}{4} + \frac{9}{6} - \frac{11}{8} + \ldots\ldots\ldots$ by Leibnitz's rule. (8)

ii) Test $\sum_{n=2}^{\infty}\frac{(-1)^n}{n(\log n)^2}$ for convergence and absolute convergence. (8)

13. a) i) Find the circle of the curvature at (0, 0) on $x + y = x^2 + y^2 + x^3$. **(8)**

ii) Find the evolute of the four cusped hypocycloid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ **(8)**

(OR)

b) i) Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$ subject to $a^n + b^n = c^n$ given c is a known constant. **(8)**

ii) Considering the evolute of a curve as the envelope of the normals, find the evolute of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **(8)**

14. a) i) If $f_1 = u - x - y - z = 0$, $f_2 = uv - y - z = 0$, $f_3 = uvw - z = 0$ then prove that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$$ **(8)**

ii) Find the Taylors series expansion for $f(x, y) = x^2 + y^2 + 2xy$ at (1, 1) upto second degree terms. **(8)**

(OR)

b) i) Find the maxima and minima of $xy(a - x - y)$. **(8)**

ii) The temperature u(x, y, z) at any point in space is $u = 400\,xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$. **(8)**

15. a) i) Change the order of integration in $I = \int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)\,dy\,dx$. **(4)**

ii) Evaluate $\iint_A (x^2+y^2)\,dx\,dy$ where A is the area bounded by the curves $x^2 = y$, $x = 1$, $x = 2$ and the x axis. **(12)**

(OR)

b) i) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ and hence evaluate $\int_0^\infty e^{-x^2}\,dx$. **(6+2)**

ii) Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(8)**